E2.5 Signals & Linear Systems

Tutorial Sheet 3 – Zero-state Responses & Convolution

(Support Lectures 4 & 5)

1.* Use direct integration, find the expression for:

a) $y(t) = u(t) * u(t)$

b) $y(t) = e^{-at}u(t) * e^{-bt}u(t)$

c) $y(t) = tu(t) * u(t)$.

2.* Use direct integration, find:

a) $y(t) = \sin t\, u(t) * u(t)$

b) $y(t) = \cos t\, u(t) * u(t)$.

3.* The unit impulse response of an LTI system is $h(t) = e^{-t}u(t)$. Use the convolution table, find this system's zero-state response $y(t)$ if the input $f(t)$ is:

a) $u(t)$

b) $e^{-2t}u(t)$

c) $\sin 3t\, u(t)$

4.** By apply the shift property of convolution, find the system's response (i.e. zero-state response) given that $h(t) = e^{-t}u(t)$ and that the input $f(t)$ is as shown in Fig 4.1.

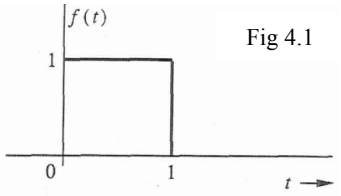


Fig 4.1

5.** A first-order allpass filter impulse response is given by

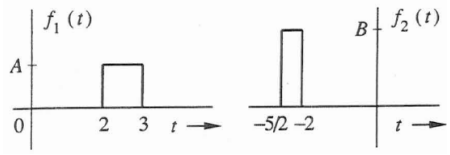
$$h(t) = -\delta(t) + 2e^{-t}u(t).$$

a) Find the zero-state response of this filter for the input $e^{t}u(-t)$.

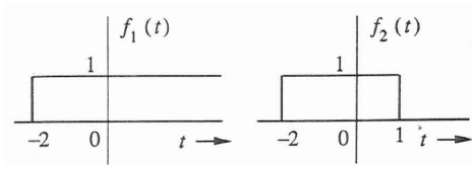
b) Sketch the input and the corresponding zero-state response.

6.** Find and sketch $c(t) = f_1(t) * f_2(t)$ for the pairs of functions shown as follow:

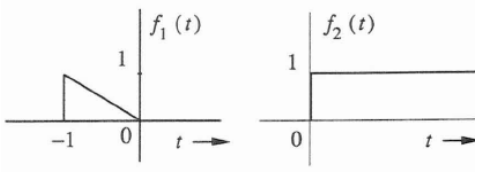
a)



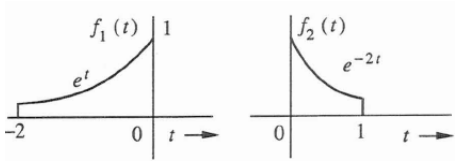
b)



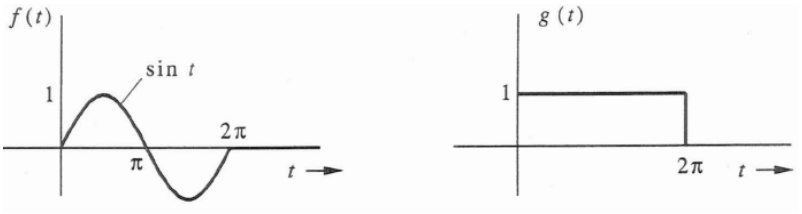
c)



d)



7.*** Find and sketch $c(t) = f(t) * g(t)$ for the pairs of functions shown below.



8.*** A line charge is located along the x axis with a charge density $f(x)$. In other words, the charge over an interval $\Delta\tau$ located at $\tau = n\Delta\tau$ is $f(n\Delta\tau)\Delta\tau$. It is also known from Coulomb's law that the electrical field $E(r)$ at a distance r from a charge q is given by:

$$E(r) = \frac{q}{4\pi\varepsilon r^2}\,,$$ where ε is a constant.

Show that electric field $E(x)$ produced by this line charge at a point x is given by

$$E(x) = f(x) * h(x)\,,$$ where $h(x) = \frac{q}{4\pi\varepsilon x^2}$.